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AFSIM - AN AIR FORCE SATELLITE INTERACTIONS MODEL.(U)
JUN 79 A MAZZELLA, E TOBENFELD, A G RUBIN F19628-76-C-0212
UNCLASSIFIED AFGL-TR-79-0138 NL

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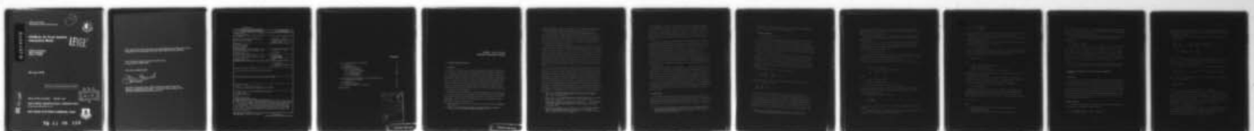

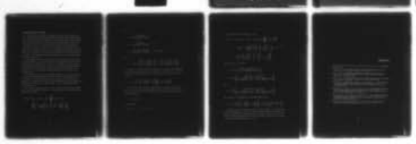

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AFGL-TR-79-0138
ENVIRONMENTAL RESEARCH PAPERS, NO. 664

AFSIM-An Air Force Satellite Interactions Model

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29 June 1979

SPACE PHYSICS DIVISION PROJECT 7661

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AIR FORCE SYSTEMS COMMAND, USAF

This report has been reviewed by the ESD Information Office (OI) and is releasable to the National Technical Information Service (NTIS).

This technical report has been reviewed and is approved for publication.

FOR THE COMMANDER

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Chief Scientist

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER AFGL-TR-79-0138, ERP-664	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) AFSIM – AN AIR FORCE SATELLITE INTERACTIONS MODEL		5. TYPE OF REPORT & PERIOD COVERED Scientific. Interim
		6. PERFORMING ORG. REPORT NUMBER ERP No. 664
7. AUTHOR(s) Andrew Mazzella, Emile Tobenfeld, Allen G. Rubin		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS Air Force Geophysics Laboratory (PHG) Hanscom AFB Massachusetts 01731		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 76610701
11. CONTROLLING OFFICE NAME AND ADDRESS Air Force Geophysics Laboratory (PHG) Hanscom AFB Massachusetts 01731		12. REPORT DATE 29 June 1979
		13. NUMBER OF PAGES 16
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

Environmental research paper

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

*RDP, Inc., Waltham, MA 02154
Work performed under Contract F19628-76-C-0212

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

SCATHA model
Computer simulation
2-D
Spacecraft charging
Satellite-plasma interactions

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

A computer program called AFSIM has been developed to conduct numerical simulations of plasma interactions with a satellite, assuming the satellite to be represented by an infinitely long cylinder and treating the plasma as discrete particles. The system is assumed to be uniform along the axis of the satellite. A number of features of the actual satellite are incorporated into the model with as much flexibility as possible to accommodate a wide range of conditions.

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Unclassified

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

409 955

Contents

AFSIM – An Air Force Satellite Interactions Model

1. A CYLINDER MODEL FOR SCATHA

1.1 Introduction

Satellites in geosynchronous orbit at approximately 6.6 earth radii have been found to charge up electrically during magnetic substorms (Deforest,[1] Rosen[2]). This charging process, called spacecraft charging, leads at times to discharges which couple radiated energy into spacecraft electronics, leading to circuit upsets, malfunction of satellites, and, on occasion, complete cessation of satellite operation. It is necessary to calculate the details of the spacecraft charging process in order to understand the physical processes taking place, to design spacecraft which are less susceptible to charging, and for the purpose of taking account of potentials in the satellite sheath in the analysis of particle and field measurements by instruments on board satellites.

The first theoretical treatment of spacecraft charging at geosynchronous orbit was carried out by DeForest,[1] employing a current balance method. The satellite is treated similarly to a Langmuir probe in a plasma. Taking into account backscattering, secondary emission, and photoemission, the steady-state floating

(Received for publication 19 June 1979)

1. DeForest, S. E. (1972) Spacecraft charging at synchronous orbit, J. Geophys. Res. 77:651.

2. Rosen, A. (1975) Spacecraft Charging: Environment Induced Anomalies, AIAA Paper 75-91, AIAA Conference, Pasadena, CA.

potential of the body in a plasma is obtained. In this method, average values of secondary emission currents and backscattered currents are employed.

For the purpose of the analysis of spacecraft charging and design, it is necessary to be able to compute a three-dimensional distribution of potentials on a satellite surface. Inouye[3] developed a circuit model, in which currents are injected at various points on a satellite surface. Approximate expressions for the currents are obtained from probe theory, and approximate expressions for secondary emission yields are employed.

The spherical satellite case has been treated in the steady state by Parker,[4] and the time-dependent case, by Rothwell et al[5] and Rothwell et al.[6] The method of Parker[4] consists of solution to the Poisson-Vlasov equations by the following means: a series of iterative operations is carried out in which electron and proton currents to the spacecraft surface are calculated first; then after the deposited surface charge has been found, the Poisson equation is solved for the potential. Currents to each surface node are obtained by tracing particle trajectories from the node into the surrounding plasma to find the proportion of trajectories which escape. The current and potential computations are iterated until a steady state results.

It is not obvious that the plasma sheath surrounding a satellite has a steady state, and in any case it is interesting to study time-dependent phenomena, particularly the emission of beams. For this reason, a time-dependent spherically symmetric description of satellite charging was carried out by Rothwell et al.[6] The method of computer simulation of plasmas is employed, in which the motion of thousands of electrons and ions are followed in their self-consistent electric field. In the spherically symmetric model, the satellite surface is considered to be covered with a single material. Backscattering and secondary emission as a function of incident particle energy are considered. In addition, photoemission is accounted for. Active control of the satellite potential is studied by simulation of the emission of charged particles from the satellite surface. In this method, an

3. Inouye, G. T. (1975) Spacecraft Potentials in a Substorm Environment, in Spacecraft Charging by Magnetospheric Plasmas, A. Rosen, Editor, MIT Press, Cambridge, MA.

4. Parker, L. W. (1975) Computer Method for Satellite Plasma Sheath in Steady-State Spherical Symmetry, AFCRL-TR-75-0410, AD A015 066.

5. Rothwell, P. L., Rubin, A. G., Pavel, A. L., and Katz, L. (1975) Simulation of the Plasma Sheath Surrounding a Charged Spacecraft, in Spacecraft Charging by Magnetospheric Plasmas, A. Rosen, Editor, MIT Press, Cambridge, MA.

6. Rothwell, P. L., Rubin, A. G., and Yates, G. K. (1977) A Simulation Model of Time-Dependent Plasma-Spacecraft Interactions, Proceedings of the Spacecraft Charging Technology Conference, C. P. Pike and R. R. Lovell, Editors, AFGL-TR-77-0051, AD A045 459.

initial configuration is set up in a spatial grid consisting of a spherical satellite, a surrounding thermal plasma, and an outer boundary. An iterative calculation is carried out, in which each of the plasma particles is first moved in the existing electric field, and the resulting charge configuration is employed as a source in the calculation of the electric field in the next time step. The computation is stepped forward in time with alternative computations of particle motion and electric field.

For the case of complex, real satellites, it is desirable to take into account the geometrical details and materials placements and properties characteristic of these vehicles. A three-dimensional quasi-static code called NASCAP, developed by Katz et al,[7] accomplishes this. The satellite is modeled by the finite-element method in a series of nested grids, each of which has a grid spacing twice as large as the previous grid. Approximately one thousand surface cells are accounted for, with fifteen materials on the spacecraft. Materials properties such as backscattering and secondary emission as a function of energy as accounted for, as is photoemission. The external magnetic field is included and satellite spin is accounted for. The photosheath is calculated, as well as charging on each satellite surface cell. Trajectories of particles from emitters on the satellite are computed as well.

Because NASCAP does not treat phenomena on the time scale of plasma oscillations and does not treat space-charge effects, either in the plasma sheath or in emitted beams, we have developed a time-dependent code in cylindrical symmetry which treats these effects. This code is designed to study beam-plasma interaction effects, active control, beam neutralization, sheath stability, space-charge effects in beam emission, and satellite surface material differential charging.

This code fills the gap between the geometrical simplicity of the spherically symmetric code and the complexity of the 3-D NASCAP code. It has the capability of handling the detailed kinetics of space-charge effects omitted from the NASCAP code and geometrical details which are unresolved in the SHEATH spherically symmetric code.

1.2 AFSIM Structure

A computer program has been developed to conduct numerical simulations of plasma interactions with a satellite, assuming the satellite to be represented by an infinitely long cylinder and treating the plasma as discrete particles. The system is assumed to be uniform along the axis of the satellite. A number of features of

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7. Katz, I., Parks, D.E., Wang, S., and Wilson, A. (1977) Dynamic Modeling of Spacecraft in a Collisionless Plasma, Proceedings of the Spacecraft Charging Technology Conference, C.P. Pike and R.R. Lovell, Editors, AFGL-TR-77-0051, AD A045 459.

the actual satellite are incorporated into the model with as much flexibility as possible to accommodate a wide range of conditions.

1.3 Features of the Model

1.3.1 PARTICLE MOTIONS

The particles are treated as discrete objects rather than as fluid elements but do not necessarily represent individual ions or electrons. The current design associates with each computer particle a cluster of identical electrons or ions so that the several million particles of the actual plasma are represented by a few thousand computer particles. In contrast to an earlier model with spherical symmetry, the parameter relating the number of real particles for each computer particle is a constant for all particles.

The particle motions are determined by the electric field produced by the satellite and by the particles themselves. The program allows the satellite to be charged to an arbitrary voltage and also allows the potential in the plasma far from the satellite to be set, thus influencing the rate at which particles emerge from the far plasma and enter the region near the satellite. The electric field is calculated from the electric potential obtained by solving Poisson's equation in a cylindrical geometry.

Thus, the equation to be solved for the potential ϕ is:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\phi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2\phi}{\partial\theta^2} = -\rho/\epsilon$$

assuming uniformity is the z-direction. With a coordinate transformation $S = \ln r$, this equation becomes:

$$\frac{\partial^2\phi}{\partial S^2} + \frac{\partial^2\phi}{\partial\theta^2} = \frac{-r^2\rho}{\epsilon} \quad ,$$

which is the form for rectangular coordinates, using a modified density $r^2\rho$.

This transformation is effected by choosing the radial gridpoints to have uniform increments in ln r, rather than as an explicit coordinate transformation. A standard Poisson-solving routine (Hockney[8]) for rectangular coordinate systems is then employed to find the potential, using the modified charge density. This approach also requires an auxiliary routine to solve for the potential on the satellite itself, to provide a necessary boundary condition for the Poisson solver. The

8. Hockney, R.W. (1970) The potential calculation and some applications, in Methods in Computational Physics, Vol. 9, Academic Press, N.Y.

auxiliary routine uses a moment series expansion to determine the satellite potential in terms of the charge distribution and the constant value assumed for the potential in the far plasma region.

No magnetic field has been incorporated into the program at this time, but it would be feasible to add a magnetic field which is uniform along the direction of the cylinder axis.

Particles are "lost" from the system by striking the surface of the satellite or escaping from the region studied, and are added to the system by a variety of emission processes on the satellite, described below, or by entering into the region studied from the surrounding plasma.

1.3.2 SECONDARY EMISSION

Ions or electrons which strike the satellite are assumed to generate secondary electrons, taking into account both the energy and angle of incidence of the impacting particle to determine the yield of secondary electrons.

The equation used to evaluate the secondary emission for incident electrons is:

$$\delta = C_o \left[\frac{1 - \exp\left(-(X_o E)^n\right)}{(X_o E)^{n-1}} \right] \exp\left(C_1 [1 - \cos\theta]\right) \quad ,$$

where

δ = number of secondary electrons

E = energy of incident electrons

θ = angle of incidence with respect to normal to surface.

The parameters C_o, C_1, and X_o can be chosen to be different over the angular sectors of the satellite to reflect the properties of different materials with respect to secondary emission.

For incident ions, the equation used is:

$$\delta = \frac{a_o \sqrt{E} \sec\theta}{1 + E/X_1}$$

where a_o and X_1 reflect properties of the materials in each angular sector of the satellite.

For either incident ions or incident electrons, the emitted secondary electrons are assumed to have a Maxwellian velocity distribution:

$$f(v) \propto v^2 \exp\left(\frac{-v^2}{2b}\right)$$

The velocity dispersion "b" for the emitted electrons can depend on the species of incident particle, and its value can easily be changed at execution time.

The direction of the velocity of the emitted particles has a cosine distribution in angle with respect to the normal.

The corresponding process for ion emission (sputtering) has not been treated, as the magnitude of that process is relatively small.

1.3.3 BACKSCATTERING

Inelastic collision of electrons with the satellite is treated as a backscattering process, again taking into account the energy and angle of incidence of the impacting particle to determine the yield. The equation used here is:

$$\beta = B_o(A_o\ E^{-m})^{\cos\theta}$$

where

β = number of backscattered electrons

E = energy of incident electrons

θ = angle of incidence with respect to normal to surface.

The parameters A_o and B_o can likewise be set to different values on the different angular sectors of the satellite.

The energy of the emitted particles is directly related to the energy of the incident particles, according to $f(v) \propto v^3/E^2$, for incident energy E, provided $m_e v^2/2 \leq E$, and again a cosine distribution in angle is assumed.

As with the sputtering process, backscattering of ions is neglected.

1.3.4 PHOTOEMISSION

The emission of photoelectrons from the satellite surface is treated in a manner similar to secondary emission, taking account of the incidence angle of solar illumination and the photoemissive coefficients of the different sectors of the satellite to determine the photoemission yield. Thus, the relevant formula is:

$$\epsilon = k_o f(\cos\alpha)$$

where

ϵ = number of photoelectrons per unit time per unit surface area

f = solar illumination flux

α = angle of solar illumination with respect to normal to surface.

The solar flux is essentially zero or one, depending upon whether the sector is in shadow or not, while k_o is a photoemission coefficient which can be set to different values for different sectors of the satellit.

Currently, the velocity spectrum of the emitted photoelectrons is assumed to be a Maxwellian with the cosine angular dependence previously assumed for secondaries.

1.3.5 PARTICLE BEAMS

Corresponding to the particle beams on the satellite, there is a routine which emits particles from a localized region of the satellite in a beam pattern. The beam pattern is given according to a $1/\theta_o \cos \theta/\theta_o$ law, for $0 \leq \theta \leq \theta_o$, and the velocity distribution of the beam particles can be chosen to be either monochromatic ($v = v_o$ for all particles) or Maxwellian ($f(v) \propto v^2 \exp(-v^2/2v_o^2)$). There are effectively separate particle beams for ions and electrons, and, in addition to θ_o and r_o, the location, aiming direction, and current for each beam can be specified.

An option is available by which the total charge emitted by the beams can be linked to the total satellite charge, thus influencing the potential on the satellite.

2. ELECTROSTATIC POTENTIAL CALCULATIONS FOR A CYLINDRICAL MODEL

2.1 Introduction

Recent investigations of spacecraft interaction with the surrounding plasma have led to the development of a two-dimensional model for numerical simulations. This model treats the spacecraft as an infinitely long cylinder and studies the motion of particles in the cylindrical annulus between the spacecraft and an arbitrary boundary at some distance into the plasma. To represent the motion of the particles properly, it is necessary to perform calculations of the electric field which is established by the distribution of charges in the region and by imposed potentials on the spacecraft. This section describes the potential calculation method that has been implemented.

2.2 Poisson's Equation

For a system with translational symmetry in the Z-direction (along the axis of the spacecraft cylinder), Poisson's equation is:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\phi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2\phi}{\partial\theta^2} = -\frac{\rho(r,\theta)}{\epsilon} \quad \text{(MKS units)}$$

where the electrostatic potential is ϕ, and ρ is the charge density (per unit area). The charges are effectively infinitely long rods in this representation and interact via a logarithmic potential.

Converting the above form of Poisson's equation to suitable form for a discrete grid gives:

$$\frac{2r_i}{D_{i-1}}\left[\frac{r_i\,\phi_{i+1,j}}{\Delta_i} - \phi_{i,j}\left(\frac{r_i}{\Delta_i} + \frac{r_{i-1}}{\Delta_{i-1}}\right) + \frac{r_{i-1}\,\phi_{i-1,j}}{\Delta_{i-1}}\right]$$

$$+ \frac{\phi_{i,j+1} - 2\phi_{i,j} - \phi_{i,j-1}}{\delta^2} = \frac{-r_i^2\,\rho_{i,j}}{\epsilon} ,$$

where $\phi_{i,j} = \phi(r_i, \theta_j)$, $\rho_{i,j} = \rho(r_i, \theta_j)$, and $\delta = \theta_j - \theta_{j-1}$, $\Delta_i = r_{i+1} - r_i$, $D_i = r_{i+2} - r_i$. For a uniform interval in the radial grid points, this form produces coefficients of $\phi_{i,j}$, $\phi_{i\pm1,j}$ which depend on i. However, if Δ_i is proportional to r_i, these coefficients are constant.

Thus, letting $\Delta_i = \alpha r_i$, so that $D_i = \alpha(2 + \alpha)r_i$, Poisson's equation becomes:

$$\frac{2(1+\alpha)}{\alpha^2(2+\alpha)}\left[\phi_{i+1,j} - 2\phi_{i,j} + \phi_{i-1,j}\right] + \frac{\phi_{i,j+1} - 2\phi_{i,j} + \phi_{i,j-1}}{\delta^2} = \frac{-r_i^2\,\rho_{i,j}}{\epsilon}$$

This formulation corresponds to a transformation of the radial coordinate according to $S = \ln r$, which gives

$$\frac{\partial^2\phi}{\partial S^2} + \frac{\partial^2\phi}{\partial\theta^2} = -\frac{\tilde{\rho}}{\epsilon} ,$$

where $\tilde{\rho} = r^2\rho$ for the transformed density in (S, θ) coordinates.

The charge density is determined from the number of charges within a particular grid cell, but each charge is treated as being uniformly distributed over a cell in (S, θ) coordinates, rather than (r, θ) coordinates. This is implemented by assuming a charge density distribution $\rho = C_{ij}/r^2$ for each cell, normalized according to $\int \rho r\, dr\, d\theta = q_{ij}$, and then using the average of $r^2\rho$ over the cell for the right side of Poisson's equation. Thus, $r^2\rho_{i,j}$ is replaced by $-q_{i,j}/[\delta \ln(1+\alpha)]$ in the formula above.

Note that different charges within a cell do not interact, and only the net charge within a cell contributes to the potential. Thus, the grid produces a potential that is both "softer" (reduced short-range interaction) and "smoother" (spatially averaged) than the actual interparticle interaction.

2.3 Boundary Conditions for the Model

In solving Poisson's equation, the boundary conditions must be specified at some radius in the far plasma (R_B) and over some surface on or within the spacecraft. The spacecraft itself is modeled as a conducting cylinder completely surrounded by a dielectric layer. This allows the charge distribution on the surface of the spacecraft to be that determined only by particle impacts and emission mechanisms, yet allows for the specification of a unique fixed potential for the spacecraft. Conceptually, therefore, the two boundaries are concentric cylinders with a constant potential on each. However, neither potential is necessarily constant in time. In particular, if the spacecraft potential is not fixed, it will "float" with respect to the outer boundary potential, depending on the charge distribution in the intervening region.

If the boundary condition is left solely in terms of the potential on the interior conducting cylinder ($r = R_i$), then the routine for solving Poisson's equation must also involve the conditions at two interfaces: the conductor/dielectric interface and the dielectric/vacuum interface. However, if the interior boundary condition is transformed into a specification of the potential on the outer surface of the spacecraft ($r = R_p$), then Poisson's equation need only be solved over a homogeneous region.

This transformation can be accomplished by solving for the surface potential for the case of an isolated charge, given the potential at the outer boundary and the specification of the potential on the interior conductor ("floating" or fixed potential). The solution for an arbitrary charge distribution is then the appropriate superposition of solutions for isolated charges. (This is essentially a Green's function approach.)

For a charge q at (r_c, θ_c), with $\phi = \phi_o$ at $r = R_i$ (the inner conducting cylinder) and $\phi = \phi_i$ at $r = R_B$ (the far plasma), the potential at (R_p, θ) (on the surface of the spacecraft) is given by:

$$\phi(R_p, \theta) = a_1 \phi_o + a_2 \phi_1 + a_3 \frac{q}{2\pi\epsilon} + \sum_{n=1}^{N/2} \cos n(\theta - \theta_c) \cdot$$

$$\left[\left(C_n + \frac{q}{2\pi n \epsilon r_c^n} \right) R_p^n - \left(C_n R_p^{2n} + \frac{q r_c^n}{2\pi n \epsilon} \right) \frac{1}{R_p^n} \right] .$$

where

$$a_1 = \frac{\kappa \ln (R_p/R_B)}{\kappa \ln (R_p/R_B) + \ln (R_i/R_p)}$$

$$a_2 = \frac{\ln (R_i/R_p)}{\kappa \ln (R_p/R_B) + \ln (R_i/R_p)} = 1 - a_1$$

$$a_3 = \frac{- \ln (r_c/R_B) \ln (R_i/R_p)}{\kappa \ln (R_p/R_B) + \ln (R_i/R_p)} = -a_2 \ln (r_c/R_B)$$

and

$$C_n = - \frac{q}{2\pi\epsilon n r_c^n} \left[\frac{(R_p^{2n} + r_c^{2n})(R_p^{2n} - R_i^{2n}) - \kappa (R_p^{2n} - r_c^{2n})(R_p^{2n} + R_i^{2n})}{(R_p^{2n} + R_B^{2n})(R_p^{2n} - R_i^{2n}) - \kappa (R_p^{2n} - R_B^{2n})(R_p^{2n} + R_i^{2n})} \right]$$

In principle, the summation should be extended to $n = \infty$, but on a discrete grid it is restricted according to the number of angular sectors N ($= 2\pi/\delta$). For the case of a floating potential on the spacecraft, with a total charge Q_i on the interior cylinder,

$$\phi_o = \phi_1 - \frac{Q_i}{2\pi\kappa\epsilon} \left[\kappa \ln \left(\frac{R_p}{R_B} \right) + \ln \left(\frac{R_i}{R_p} \right) \right] - \frac{q}{2\pi\epsilon} \ln \left(\frac{r_c}{R_B} \right)$$

Some modifications to the above expressions are required to accommodate the convention of having the charges distributed over each cell, rather than as discrete charges. This is accomplished by a superposition of the above solutions for a charge density

$$\rho = \frac{q}{\delta \ln (1 + \alpha) r^2}$$

over the region

$$\frac{r_c}{1 + \alpha} < r \le r_c \quad , \quad \theta_c \le \theta < \theta_c + \delta \quad .$$

This produces a potential $\tilde{\phi}(R_p, \theta)$, with:

$$\tilde{\phi}(R_p, \theta) = a_1\phi_o + a_2\phi_1 + b_1 \frac{q}{2\pi\epsilon} + \frac{q}{2\pi\epsilon\,\delta\,\ln(1+\alpha)} \sum_{n=1}^{N/2} \frac{2}{n^3} \sin\left(\frac{n\delta}{2}\right)$$

$$\cos n\left(\theta - \theta_c - \frac{\delta}{2}\right)\left\{\left(\frac{R_p}{r_c}\right)^n \left[A_n\left(1 - \frac{R_B^{2n}}{R_p^{2n}}\right) - 1\right]\left[1 - (1+\alpha)^n\right]\right.$$

$$\left. + \left(\frac{r_c}{R_p}\right)^n \left[B_n\left(\frac{R_B^{2n}}{R_p^{2n}} - 1\right) - 1\right]\left[1 - \frac{1}{(1+\alpha)^n}\right]\right\},$$

where a_1 and a_2 are as before,

$$b_1 = \ln(R_p/R_i)\left[\frac{\ln(r_c/R_B) - \frac{1}{2}\ln(1+\alpha)}{\kappa\ln(R_p/R_B) + \ln(R_i/R_p)}\right]$$

$$A_n = R_p^{2n}\left[\frac{(R_p^{2n} - R_i^{2n}) - \kappa(R_p^{2n} + R_i^{2n})}{(R_p^{2n} + R_B^{2n})(R_p^{2n} - R_i^{2n}) - \kappa(R_p^{2n} - R_B^{2n})(R_p^{2n} + R_i^{2n})}\right]$$

and

$$B_n = R_p^{2n}\left[\frac{(R_p^{2n} - R_i^{2n}) + \kappa(R_p^{2n} + R_i^{2n})}{(R_p^{2n} + R_B^{2n})(R_p^{2n} - R_i^{2n}) - \kappa(R_p^{2n} - R_B^{2n})(R_p^{2n} + R_i^{2n})}\right]$$

The condition for a floating spacecraft potential then becomes

$$\phi_o = \phi_1 - \frac{Q_i}{2\pi\kappa\epsilon}\left[\kappa\ln\left(\frac{R_p}{R_B}\right) + \ln\left(\frac{R_i}{R_p}\right)\right] - \frac{q}{2\pi\epsilon}\left[\ln\left(\frac{r_c}{R_B}\right) - \frac{\ln(1+\alpha)}{2}\right]$$

The solution is now completely formulated, given a method of solving Poisson's equation in rectangular coordinates. For this particular application, Hockney's[8] Fourier Analysis/Cyclic Reduction Method was employed. The Fourier transform routine for Hockney's programs is also used to obtain the superposition of solutions for setting the boundary potentials at $r = R_p$.

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